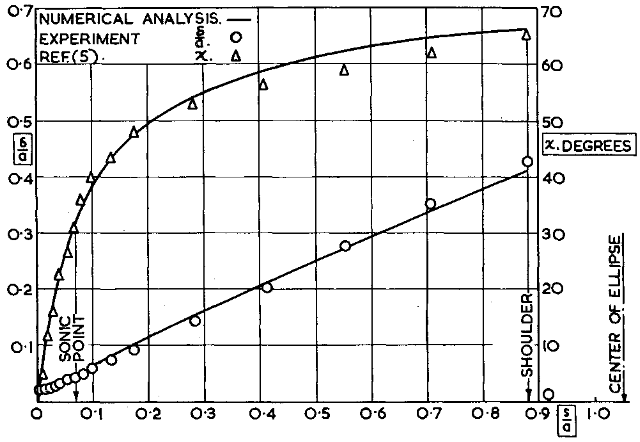

Fig. 4 Comparison of theoretical and experimental shock shape parameters δ and χ on elliptic nose, $a/b = 5$, $M_\infty = 6.8$, $RN_\infty = 6.72 \times 10^4$/in.

convergence of the solution if the length from stagnation point to sonic point was divided into 200 or more steps.

The axis ratio of the ellipse a/b was varied from 0.5 to 10, and Mach numbers were 3, 6.8, and ∞. The results for shock standoff distance (Table 1) are rounded off at four significant figures, but in many cases were carried to eight places. Figure 2 shows that at hypersonic speeds the stagnation-point shock standoff distance depends only upon the nose radius of curvature for axis ratios above 4 or 5. The limiting value appears to be about 0.46 of the nose radius of curvature. A cross-plot of Belotserkovskii's results for axisymmetric ellipsoids[4] is included for comparison.

Experimentation[5] at $M_\infty = 6.8$ and Reynolds number $RN_\infty = 6.7 \times 10^4$/in. on an elliptic cylinder having $a/b = 5$ shows agreement with the preceding solution for shock standoff distance in the region between the stagnation point and sonic point within the limit of error of measurement. This was 7% on δ/R at the stagnation point, falling to 4% at the sonic point. Figures 3 and 4 show the experimental shock shape compared with one-strip theory for the elliptic cylinder.

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Technical Comments

Comment on "Mass and Magnetic Dipole Shielding against Electrons of the Artificial Radiation Belt"

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THE article by Bhattacharjie and Michael[1] shows that magnetic radiation shielding against electrons of the artificial belt is attractive when compared to solid shielding. It appears that they have chosen a relatively unfavorable configuration for their magnetic shield; as a result of this, they have emphasized a relatively unimportant aspect of the design, and substantially understated the attractiveness of their concept.

The shielded volume V around a dipole is proportional to $M^{3/2}$, where M is the magnetic moment of the dipole. M, in turn, is proportional to Ia^2 where I is the total circulating current and a the radius of the coil. The mass of structural material m_{st} required to contain the magnetic stresses is[2–4] proportional to the magnetic energy stored by the device; this varies as I^2a for fixed geometrical ratios. For a given maximum current density of the superconducting material, the mass of superconductor m_{sc} required is proportional to Ia. Thus, for a given shielded volume (or magnetic moment), both masses can be indefinitely reduced by reducing I and increasing a. Bhattacharjie and Michael have worked, however, at a fixed magnetic field strength of 50 kgauss. That is, they have fixed the ratio I/a. From the foregoing, it can be seen that this implies that $m_{st} \propto V^{2/3}$, and this proportionality appears in their Eq. (6) and also in their Fig. 2. Much lower weights than they quote can be achieved by reducing the level of both the current and magnetic field, and by enlarging the geometry. The optimization actually performed by Bhattacharjie and Michael deals only with the aspect ratio of the solenoid providing the magnetic moment. Based on considerations of structural efficiency, they arrive at the conclusion that the optimum aspect ratio is roughly square. We shall see below that structural considerations are, in any event, unimportant for the shielding purposes under consideration.

One cannot, of course, achieve indefinitely lower values for m_{st} and m_{sc} by reducing I and increasing a. The shielded volume declines sharply if a is larger than the Stormer radius c_{st} because the magnetic field in the shielded volume becomes significantly different from the field due to a dipole. Minimum weight for a given shielded volume occurs when these lengths are roughly equal, thus

$$a/c_{st} = (4\pi\ pa^2/eM)^{1/2} = (4p/\mu_0 eI)^{1/2} \approx 1$$

in mks units. For 10 Mev electrons, the momentum to charge ratio p/e is 0.035. Therefore, a shield of any size designed to protect against 10 Mev electrons should operate with fixed total current of about 10^5 amp.

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Since $m_{st} \propto I^2a$, and $m_{sc} \propto Ia$, then, whatever the size, there is a critical current below which $m_{st} < m_{sc}$. Using the structural material chosen by Bhattacharjie and Michael and a critical current density of 10^5 amp/cm^2, this current is roughly 10^7 amp. Since this is 100 times greater than the current required for shielding, we expect the structural weight to be about 1% of the superconducting material weight, independent of the size. The whole question of structure is, therefore, unimportant in the context of 10 Mev electrons.

Using the value $a/c_{st} = 0.6$ taken from Ref. 2 and using the same reference to estimate the true shielded volume for a coil of this type, one arrives at the conclusion that the weight of the shield lies almost entirely in the superconducting material; this weight is roughly given by

$$m_{sc} = 25V^{1/3}$$

The structural weight is a negligible fraction of the superconducting weight. Note the correct power dependence of m on V, when the optimization is done at constant current. These weights are lower than those quoted by Bhattacharjie and Michael by a factor of 3 when $V = 10^2$, and by a factor of 50 when $V = 10^5$. With weights as low as these, it is likely that considerations of the area of cryogenic surface required would exert a significant influence on the design.

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Comments on "Electron Fluctuations in an Equilibrium Turbulent Plasma"

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THE analysis according to which Demetriades[1] obtains expressions for the "proper" average electron density and the rms fluctuation of electron density for a turbulent plasma in thermochemical equilibrium includes an assumption that implies a particular probability distribution for the temperature. As will be shown, this distribution consists of two delta functions, each one being displaced from the average temperature by the rms temperature fluctuation. Thus a special case of the "marble cake" model[3, 4] is implied by Demetriades' assumption.

The probability distribution can be derived according to standard methods as follows. Let $P(T)$ be the probability distribution in T space; then the expected value $\langle f(T)\rangle$ of any function $f(T)$ is

$$\langle f(T)\rangle = \int_0^\infty f(T)\, P(T)\, dT \tag{1}$$

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Now consider the Fourier cosine transform of $P(T)$

$$P^*(\lambda) = \int_0^\infty P(T) \cos\lambda T\, dT \tag{2}$$

and let

$$\bar{T} = \langle T\rangle = \int_0^\infty TP(T)\, dT$$

Then Eq. (2) may be written as

$$\begin{aligned} P^*(\lambda) &= \int_0^\infty P(T) \cos\lambda(T - \bar{T} + \bar{T})\, dT \\ &= \int_0^\infty P(T)[\cos\lambda(T-\bar{T})\cos\lambda\bar{T} - \sin\lambda(T-\bar{T}) \times \sin\lambda\bar{T}]dT \\ &= \int_0^\infty P(T)\left\{\cos\lambda\bar{T}\left[1 - \frac{\lambda^2(T-\bar{T})^2}{2!} + \frac{\lambda^4(T-\bar{T})^4}{4!} + \ldots\right] - \sin\lambda\bar{T}\left[\frac{\lambda(T-\bar{T})}{1!} - \frac{\lambda^3(T-\bar{T})^3}{3!} + \ldots\right]\right\} dT \end{aligned}$$

Therefore,

$$P^*(\lambda) = \cos\lambda\bar{T}[1 - (\lambda^2/2!)\langle(T-\bar{T})^2\rangle + \ldots] - \sin\lambda\bar{T}[-(\lambda^3/3!)\langle(T-\bar{T})^3\rangle + \ldots] \tag{3}$$

If one now invokes Demetriades'[1] assumptions, viz.,

$$\langle(T-\bar{T})^m\rangle = \begin{cases} 0 & m = \text{odd integer} \\ [\langle(T-\bar{T})^2\rangle]^{m/2} & m = \text{even integer} \end{cases}$$

then Eq. (3) becomes

$$\begin{aligned} P^*(\lambda) &= \cos\lambda\bar{T}\left[1 - \frac{\lambda^2}{2!}\langle(T-\bar{T})^2\rangle + \frac{\lambda^4}{4!}\langle(T-\bar{T})^2\rangle^2 + \ldots\right] \\ &= \cos\lambda\bar{T}\cos\lambda[\langle(T-\bar{T})^2\rangle]^{1/2} \end{aligned} \tag{4}$$

Inverting, there results for $P(T)$

$$P(T) = \frac{2}{\pi}\int_0^\infty \cos\lambda\bar{T}\cos\lambda[\langle(T-\bar{T})^2\rangle]^{1/2}\cos\lambda T\, d\lambda$$

or

$$P(T) = \frac{1}{\pi}\int_0^\infty \cos\lambda T\,(\cos\lambda\{\bar{T} + [\langle(T-\bar{T})^2\rangle]^{1/2}\} + \cos\lambda\{\bar{T} - [\langle(T-\bar{T})^2\rangle]^{1/2}\})\, d\lambda \tag{5}$$

Therefore,[2]

$$P(T) = \tfrac{1}{2}[\delta(T - \{\bar{T} + [\langle(T-\bar{T})^2\rangle]^{1/2}\}) + \delta(T - \{\bar{T} - [\langle(T-\bar{T})^2\rangle]^{1/2}\})] \tag{6}$$

Thus, if physical significance is to be assigned to the results of (1), then only the temperatures $\bar{T} \pm [\langle(T-\bar{T})^2\rangle]^{1/2}$ are allowed to occur in the physical problem. This is physically unrealistic, and, accordingly, quantitative information based on the results of (1) should be used with caution. However, it does correspond to the special case of the marble-cake model in which equal weightings occur for "hot" and "cold" constituents.

If a distribution such as Eq. (6) is accepted, then all statistical properties of any function of T (i.e., expected values, rms deviations therefrom, etc.) may be immediately evaluated without recourse to further approximations. For example, applying Eq. (6) to electron density n_e as a function of temperature T, there results for the average electron density

$$\frac{\langle n_e(T)\rangle}{n_e(\langle T\rangle)} = \frac{\overline{n_e(T)}}{n_e(\bar{T})} = \frac{1}{2}\left(\frac{n_e\{\bar{T} + [\overline{(\Delta T)^2}]^{1/2}\} + n_e\{\bar{T} - [\overline{(\Delta T)^2}]^{1/2}\}}{n_e(\bar{T})}\right) \tag{7}$$